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(54) Title: A METHOD AND SYSTEM TO SOLVE DYNAMIC MULTI-FACTOR MODELS IN FINANCE

(57) Abstract: Methods and systems for estimating time-varying factor exposures of either an individual financial instrument or a portfolio of such instruments, through the solution of a constrained multi-criteria dynamic optimization problem, providing an estimation error function and one or more transition error functions to be minimized over a period of time. The factor exposures relay the influence of the factors on the return of the instrument or portfolio. The estimation error function provides the estimation error at each time interval between the return of the asset collection and a sum of products of each factor exposure and its respective factor. Each transition error function provides a transition error of each factor exposure between time intervals. In one embodiment, the constraints can include a budget constraint and non-negativity bounds applying to some or all of the factor exposures. In other embodiments, the method and system can be applied to estimating any time-varying weight that is used in a model, to relay the influence of one or more independent variables on a dependent financial or economic variable, through the solution of a constrained multi criteria dynamic problem, minimizing estimation error and transition error terms. In other embodiments, the solution of a multi criteria dynamic problem can be used as part of a method and system to determine structural breakpoints for each factor, an also as part of a method and system for determining optimal parameters to weight the transition error functions and selecting the factors included in the model.

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A Method and System to Solve Dynamic Multi-Factor Models in Finance

RELATED APPLICATION INFORMATION

This application claims the benefit of U.S. Provisional Application No. 60/378,562 filed on May 7, 2002. U.S. Provisional Application No. 60/378,562 is expressly incorporated herein by reference in its entirety into this application.

FIELD OF THE INVENTION

The present invention relates generally to systems and methods for estimating time-varying factor exposures in financial or economic model or problem, through the solution of a multi-factor dynamic optimization of the model or problem, while meeting the constraints for the estimated time-varying factor exposures in the model or problem.

BACKGROUND OF THE INVENTION

The following references, discussed and/or cited in this application, are hereby expressly incorporated herein by reference in their entirety into this application:

1. Sharpe, William F., Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance*, Sept. 1964;
2. Chen, Nai-fu, Roll, Richard, Ross, Stephen A., Economic forces and the stock market, *Journal of Business*, 59, July 1986;
3. Rosenberg, B., Choosing a multiple factor model, *Investment Management Review*, November/December 1987;
4. Sharpe, William F., Determining a Fund's Effective Asset Mix, *Investment Management Review*, November/December 1988;
5. Sharpe, William F., Asset Allocation: Management Style and Performance Measurement, *The Journal of Portfolio Management*, Winter 1992;
6. Kalaba, R., Tesfatsion, L., Time-Varying Linear Regression via Flexible Least Squares, *Computers and Mathematics with Applications*, 17, 1989;
7. Kalaba, R., Tesfatsion, L., Flexible least squares for approximately linear systems, *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-5, 1990;
8. Tesfatsion, L., GFLS implementation in FORTRAN and the algorithm, http://www.econ.iastate.edu/tesfatsi/gflshelp.htm (1997);

9. Lütkepohl, H., Herwartz, H., Specification of varying coefficient time series models via generalized flexible least squares, Journal of Econometrics, 70, 1996;

10. Wright, S., Primal-dual interior-point methods, *SIAM*, 1997; and

11. Stone, M., Cross-validatory choice and assessment of statistical predictions, *Journal of Royal Statistical Soc.*, B 36, 1974.

A. Multi-factor models in finance

Factor models are well known in finance, among them a multi-index Capital Asset Prices Model (CAPM) and Arbitrage Pricing Theory (APT). These models allow for a large number of factors that can influence securities returns.

The multi-factor CAPM, for example, described in Sharpe, William F., Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance*, Sept. 1964, pp. 425-442, can be represented by the equation:

$$r - r^{(f)} \cong \alpha + \beta^{(1)}(r^{(1)} - r^{(f)}) + \beta^{(2)}(r^{(2)} - r^{(f)}) + \ldots + \beta^{(n)}(r^{(n)} - r^{(f)}) \qquad (1)$$

where r is the investment return (security or portfolio of securities), $r^{(i)}$ are returns on the market portfolio as well as changes in other factors like inflation, and $r^{(f)}$ is return on a risk-free instrument.

In the multi-factor APT model (described, for example, in Chen, N., Richard R., Stephen A. R., Economic forces and the stock market, *Journal of Business*, 59, July 1986, pp. 383-403):

$$r \cong \alpha + \beta^{(1)} f^{(1)} + \beta^{(2)} f^{(2)} + \ldots + \beta^{(n)} f^{(n)}, \qquad (2)$$

the factors $f^{(i)}$ are typically chosen to be the major economic factors that influence security returns, like industrial production, inflation, interest rates, business cycle, etc. (described, for example, in Chen, N., Richard R., Stephen A. R., Economic forces and the stock market *Journal of Business*, 59, July 1986, pp. 383-403, and in Rosenberg, B., Choosing a multiple factor model, *Investment Management Review*, November/December 1987, pp. 28-35).

Coefficients $\beta^{(1)},\ldots,\beta^{(n)}$ in the CAPM (1) and APT (2) models are called factor exposures. Along with the constant α, the factor exposures make the vector of model parameters

$(\alpha, \beta^{(1)}, \ldots, \beta^{(n)})$, which is typically estimated by applying a linear regression technique to the time series of security/portfolio returns r_t and economic factors $r_t^{(i)}$ or $I_t^{(i)}$ over a certain estimation window $t = 1, \ldots, N$:

$$(\hat{\alpha}, \hat{\beta}^{(1)}, \ldots, \hat{\beta}^{(n)}) = \underset{\alpha, \beta^{(1)}, \ldots, \beta^{(n)}}{\arg\min} \sum_{t=1}^{N} (r_t - \alpha - \beta^{(1)} I_t^{(1)} - \ldots - \beta^{(n)} I_t^{(n)})^2 . \qquad (3)$$

One of the most effective multi-factor models for analyses of investment portfolios, called the Returns Based Style Analysis (RBSA), was suggested by Prof. William F. Sharpe (for example, in Sharpe, William F., Determining a Fund's Effective Asset Mix, *Investment Management Review*, November/December 1988, pp. 59-69, and in Sharpe, William F., Asset Allocation: Management Style and Performance Measurement, *The Journal of Portfolio Management*, Winter 1992, pp. 7-19). In the RBSA model, the periodic return y of a portfolio consisting of n kinds of assets is approximately represented by a linear combination of single factors $(x^{(1)}, \ldots x^{(n)})$ whose role is played by periodic returns of generic market indices for the respective classes of assets. To enhance the quality of parameter estimation, a set of linear constraints is added to the basic equation:

$$\begin{aligned} & y \cong \alpha + \beta^{(1)} x^{(1)} + \beta^{(2)} x^{(2)} + \ldots + \beta^{(n)} x^{(n)}, \\ & \textstyle\sum_{i=1}^{n} \beta^{(i)} = 1, \ \beta^{(i)} \geq 0, \ i = 1, \ldots, n. \end{aligned} \qquad (4)$$

In such a model, $x^{(i)}$, $i = 1, \ldots, n$, represent periodic returns (for example, daily, weekly or monthly) of generic market indices such as bonds, equities, economic sectors, country indices, currencies, etc. For example (as described in Sharpe, William F., Asset Allocation: Management Style and Performance Measurement, *The Journal of Portfolio Management*, Winter 1992, pp. 7-19), twelve such generic asset indices are used to represent possible areas of investment.

To estimate the parameters of equation (4), Sharpe used the Constrained Least Squares Technique, i.e., the parameters are found by solving the constrained quadratic optimization problem in a window of $t = 1, \ldots, N$ time periods in contrast to the unconstrained one (3):

$$
\begin{cases}
(\hat{\alpha},\hat{\beta}^{(1)},\ldots,\hat{\beta}^{(n)}) = \underset{\alpha,\beta^{(1)},\ldots,\beta^{(n)}}{\arg\min} \sum_{t=1}^{N} (y_t - \alpha - \beta^{(1)}x_t^{(1)} - \ldots - \beta^{(n)}x_t^{(n)})^2, \\
\text{subject to } \sum_{i=1}^{n} \beta^{(i)} = 1,\ \beta^{(i)} \ge 0,\ i = 1,\ldots,n.
\end{cases} \tag{5}
$$

Model parameters $(\alpha,\beta^{(1)},\ldots,\beta^{(n)})$ estimated using unconstrained (3) and constrained least squares techniques (5) represent average factor exposures in the estimation window – time interval $t = 1,\ldots,N$. However, the factor exposures typically change in time. For example, an active trading of a portfolio of securities can lead to significant changes in its exposures to market indices within the interval. Detecting such dynamic changes, even though they happened in the past, represents a very important task.

In order to estimate dynamic changes in factor exposures, a moving window technique is typically applied. For example, in RBSA model (4), the exposures at any moment of time t are determined on the basis of solving (5) using a window of K portfolio returns $[t-(K-1),\ldots,t]$ and the returns on asset class indices over the same time period (as described, for example, in 5. Sharpe, William F., Asset Allocation: Management Style and Performance Measurement, *The Journal of Portfolio Management*, Winter 1992, pp. 7-19):

$$
\begin{cases}
(\hat{\alpha}_t,\hat{\beta}_t^{(1)},\ldots,\hat{\beta}_t^{(n)}) = \underset{\alpha,\beta^{(1)},\ldots,\beta^{(n)}}{\arg\min} \sum_{s=0}^{K-1} (y_{t-s} - \alpha - \beta^{(1)}x_{t-s}^{(1)} - \ldots - \beta^{(n)}x_{t-s}^{(n)})^2, \\
\text{subject to } \sum_{i=1}^{n} \beta^{(i)} = 1,\ \beta^{(i)} \ge 0,\ i = 1,\ldots,n,
\end{cases} \tag{6}
$$

By moving such estimation window forward period by period, dynamic changes in factor exposures can be approximately estimated.

The moving window technique described above has its limitations and deficiencies. The problem setup assumes that exposures are constant within the window, yet it is used to estimate their changes. Reliable estimates of model parameters can be obtained only if the window is sufficiently large which makes it impossible to sense changes that occurred within a day or a month, and, therefore, such technique can be applied only in cases where parameters do not show marked changes within it: $(\alpha_s,\beta_s^{(1)},\ldots,\beta_s^{(n)}) \equiv \text{const}$, $t-(K-1) \le s \le t$. In addition, such approach fails to identify very quick, abrupt changes in investment portfolio exposures that can occur due to trading.

In situations, where detecting dynamic exposures represents an important task, the widow technique is inadequate, and a fundamentally new approach to estimating multi-factor models with changing properties are required. It is just the intent of this patent to fill in this gap.

B. <u>The dynamic RBSA model</u>

The multi-factor RBSA model (4), as well as the CAPM (1) and APT ones (2), are, in their essence, linear regression models with constant regression coefficients $(\alpha, \beta^{(1)}, ..., \beta^{(n)})$.

In order to monitor a portfolio for quick changes in investment allocation or investment style, deviations from investment mandate, etc., a dynamic regression RBSA model is needed to represent the time series of portfolio returns y_t as dynamically changing linear combination of a finite number n of time series of basic factors $\mathbf{x}_t = (x_t^{(1)}, ..., x_t^{(n)})^T$ with unknown real-valued factor exposures $\beta_t = (\beta_t^{(1)}, ..., \beta_t^{(n)})^T$ and an unknown auxiliary term α_t. However, in the RBSA model, both the factor exposures and the intercepts are subject to appropriate constraints $(\alpha_t, \beta_t) \in Z$, in the simplest case, the linear ones $\sum_{i=1}^{n} \beta_t^{(i)} = 1$, $\beta_t^{(i)} \geq 0$.

$$\begin{cases} y_t = \alpha_t + \sum_{i=1}^{n} \beta_t^{(i)} x_t^{(i)} + e_t = \alpha_t + \beta_t^T \mathbf{x}_t + e_t, \\ (\alpha_t, \beta_t) \in Z, \end{cases} \qquad (7)$$

where e_t is the residual model inaccuracy treated as white noise.

Note that unlike (5) and (6), the model (7) assumes that factor exposures are changing in every period or time interval t. The present invention specifies constraints $(\alpha_t, \beta_t) \in Z$ adequate to most typical problems of financial management, and describes a general way of estimating dynamic multi-factor models under those constraints.

C. <u>Insufficiency of existing methods of estimating dynamic linear models</u>

i. <u>Flexible Least Squares (FLS)</u>

A method of unconstrained parameter estimation in dynamic linear regression models was suggested by Kalaba and Tesfatsion under the name of Flexible Least Squares (FLS) method, as described, for example, in Kalaba, R., Tesfatsion, L., <u>Time-Varying Linear Regression via</u>